Numerical calculation of groundwater geofiltration processes in multilayer porous media

Jamoljon Djumanov[1], Khojiakbar Egamberdiev[2], Bakhtiyor Murodullaev[3], Dilobar Haqnazarova[4]

[1]Tashkent University of Information Technology named after Muhammad al-Khwarizmi, Tashkent, Uzbekistan

[2]Karshi Branch of the Tashkent University of Information Technology named after Muhammad al-Khwarizm, Karshi, Uzbekistani

[3]Tashkent International University of Education, Tashkent, Uzbekistan

[4]Research Institute for the Development of Digital Technologies and Artificial Intelligence, Tashkent, Uzbekistan

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Corresponding authors: Bakhtiyor Murodullaev bmurodullayev1114@gmail.com

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Abstract. The article presents the modeling of multilayer hydrogeological processes in porous media, the results of fundamental and applied research, numerical modeling of hydrogeological processes, computational mathematics and methods of finite-difference schemes for solving simple differential equations, the development and improvement of mathematical models, and considers computational algorithms and software tools for solving problems analysis and forecasting of processes.

Keywords: *balance equation, groundwater, mathematical model, geofiltration processes, hydrogeological systems, water level, filtration coefficient, infiltration, initial and boundary conditions.*

Introduction

When solving hydrogeological and land reclamation issues, depending on the changing conditions of each region, it is necessary to take into account the characteristics of irrigated lands for a particular situation, changes in the types of crops and their irrigation rates, irrigation regime and salinity. In addition, it is necessary to take into account irrigation networks, drainage networks, natural drainage networks and their interaction with groundwater, the quantitative values of groundwater inflows and outflows, as well as the design of water intake structures and their actual value withdrawal.

Approbation of theoretical, technological, methodological and software developments on drinking water supply for reliability and accuracy, creation of mathematical and simulation models for solving hydrogeological and engineering-geological problems in hydrogeological systems based on various applied methods, natural geological and mathematical modeling of geofiltration processes taking into account hydrogeological conditions, in turn, attention should be paid to the formulation and solution of specific problems that take into account technogenic conditions.

When zoning groundwater according to the conditions of hydrogeological and reclamation drainage, the following main requirements were taken into account: lithological-fascial structure and conditions for the formation of permeable and water-resistant complexes; groundwater and related equalization conditions; filtration properties in complexes of multilayer porous media; water permeability properties; distribution of planned geological boundaries (interaction of low permeable rocks); the state of the hydrographic network and its interaction with groundwater; in the tectonic structure (fractures, depressions, uplifts), two hydrodynamic zones are distinguished, taking into account regional factors.

Particular attention is paid to issues such as key indicators of interaction with groundwater, flow rates, leakage rates and identification of water resources. When considering the processes of exploration of groundwater Karshi, processing and analysis of the results of regime observations, studying and protecting the state of groundwater, predicting changes in groundwater levels, detecting leaks, studying their volume, issues of river interaction with groundwater. Mathematical modeling of underground and pressure waters, their interrelations, water exchange processes and geofiltration processes is also presented.

Main part. Groundwater balance equation - a mathematical model of geofiltration processes of hydrogeological systems in area *G* - is based on a system of differential equations representing the dynamics of groundwater runoff in a territorial plane that connects aquifers and depends on time, and has the following form: mathematical model [6; 14b, 104, 2278 b]:

$$\mu\frac{\partial h}{\partial t}=\frac{\partial}{\partial x}\left(kh\frac{\partial h}{\partial x}\right)+\frac{\partial}{\partial y}\left(kh\frac{\partial h}{\partial y}\right)+\eta\left(J-Q_b-f+Q_r-Q_d\right) \qquad (10)$$

initial condition looks like this:

$$h(x,y,t)=\phi_1(x,y); \quad (x,y)\in\Gamma_1; \quad t=t_0; \qquad (2)$$

and boundary conditions,

$$\begin{aligned} &h(x,y,t)=\phi_2(x,y); && (x,y)\in\Gamma_1; && t>t_0;\\ &-kh\frac{\partial h}{\partial n}=\phi_3(x,y); && (x,y)\in\Gamma_2 && t>t_0;\\ &-kh\frac{\partial h}{\partial n}=\gamma(h_0-h); && (x,y)\in\Gamma_3 && t>t_0 \end{aligned} \qquad (3)$$

where μ - is the ability of the layer to release water or become unsaturated (dimensionless value);

x, y - coordinates in the plane, m;

- time, day;

$h = h(x, y, t)$ - water level from the ground to the surface, m;

- formation permeability coefficient, i.e. filtration coefficient, m/day;

η – coefficient of transformation of the model into a dimensional form (coefficient of mass transfer of equations);

$J = J(x, y, t)$ - surface water infiltration, i.e. precipitation infiltration, m/day; - flooding, i.e. groundwater leakage;

- hydrogeological state of the interdependence of ground and surface waters.

Equation (1) based on the initial and boundary conditions (2) - (5) based on the developed numerical methods of F.B. Abutaliev, I.I. Izmailov and I. Khabibullaev. Scientists such as A.A. Samarsky, M.M. Krylov, S.F. Averyanov, using a one-dimensional scheme based on the numerical difference scheme and sweep methods, developed a computational algorithm that implements equation (1).

In geofiltration of groundwater and the interaction of groundwater with low pressure water is expressed using the Boussinesco equation as follows:

$$\left.\begin{aligned} \mu\frac{\partial h}{\partial t}&=\frac{\partial}{\partial x}(k_1h\frac{\partial h}{\partial x})+\frac{\partial}{\partial y}(k_1h\frac{\partial h}{\partial y})-k_{\hat{a}}(1-\frac{H}{h})+\eta W,\\ \mu*\frac{\partial H}{\partial t}&=\frac{\partial}{\partial x}(k_2m\frac{\partial H}{\partial x})+\frac{\partial}{\partial y}(k_2m\frac{\partial H}{\partial y})+k_{\hat{a}}(1-\frac{H}{h})-\eta W_1. \end{aligned}\right\} \qquad (6)$$

(6) The system is solved based on the following initial and boundary conditions: initial conditions:

$$h\big|_{t=0}=h_0, \quad H\big|_{t=0}=H_0, \qquad (7)$$

border conditions:

$$m\frac{\partial h}{\partial x}\Big|_{x=0}=-(h-h_0), \quad m\frac{\partial h}{\partial x}\Big|_{x=L}=(h-h_0), \qquad (8)$$

$$m\frac{\partial h}{\partial y}\Big|_{y=0} = -(h-h_0),\quad m\frac{\partial h}{\partial y}\Big|_{y=L} = (h-h_0),\quad (9)$$

$$m\frac{\partial H}{\partial x}\Big|_{\partial=0} = -(H-H_0),\quad m\frac{\partial H}{\partial x}\Big|_{\partial=L} = (H-H_0),\quad (10)$$

$$m\frac{\partial H}{\partial y}\Big|_{y=0} = -(H-H_0),\quad m\frac{\partial H}{\partial y}\Big|_{y=L} = (H-H_0),\quad (11)$$

$$H\Big|_{x=m+0} = h\Big|_{x=m-0},\quad H\Big|_{y=m+0} = h\Big|_{y=m-0},\quad (12)$$

$$k_2 m\frac{\partial H}{\partial x}\Big|_{x=m+0} = k_1 m\frac{\partial h}{\partial x}\Big|_{x=m-0},$$

$$k_2 m\frac{\partial H}{\partial y}\Big|_{y=m+0} = k_1 m\frac{\partial h}{\partial y}\Big|_{y=m-0}.\quad 13)$$

where h_0, H_0 - initial values of groundwater levels and pressures.

The boundary conditions are set on the basis of three types of expressions I, II, III, which determine the relationship between the groundwater level or water consumption, or the level and water consumption at the boundary of the geofiltration area, depending on natural, hydrological and hydrogeological conditions. Under the conditions of the above models and their implementation using methods, it is possible to study changes in the state of groundwater, their relationship with surface water, as well as changes in freshwater in the plane and in time.

On a regional scale, groundwater flows along the riverbed for tens of kilometers. The patterns of regional formation of groundwater are determined by the geological structure, structural features of aquifers, water availability of underground runoff and water consumption conditions. High-quality modeling takes into account the features of calculation processes and the focus of factors on the natural state (adequacy), knowledge programming and complex hydrogeological conditions.

A numerical algorithm based on the finite difference method has been developed for the numerical integration of the expression of problems (6) - (16), represented by systems of nonlinear differential equations with a specific product.

In the numerical solution, studies were carried out on the introduction of variables, the use of indefinite difference schemes, approximations, reduction to a system of algebraic equations, the solution of a system of linear algebraic equations by the sweep method, as well as finding in the directions *Ox* and *Oy* the boundaries of the value of groundwater levels.

Using the finite-difference approach to problems, we create a system of algebraic equations, solving it, determine the required parameters of the object and the optimal values of their change in time and direction. To solve problems (1) - (5), we introduce dimensionless quantities and, replacing the differential operators in equation (1) with finite-difference operators, using the scheme of the longitudinal-transverse direction, we obtain the following in the *Ox* direction:

$$\frac{1}{\tilde{h}_{i,j}}\frac{(h^2)^{n+\frac{1}{2}}_{i,j}-(h^2)^n_{i,j}}{0.5\Delta\tau} = \frac{k_{i-0.5,j}(h^2)^{n+\frac{1}{2}}_{i-1,j}-(k_{i-0.5,j}+k_{i+0.5,j})(h^2)^{n+\frac{1}{2}}_{i,j}+}{\Delta x^2}+\frac{k_{i+0.5,j}(h^2)^{n+\frac{1}{2}}_{i+1,j}}{\Delta x^2}+$$
$$+\frac{k_{i,j-0.5}(h^2)^n_{i,j-1}-(k_{i-0.5,j}+k_{i+0.5,j})(h^2)^n_{i,j}}{\Delta y^2}+\frac{k_{i+0.5,j}(h^2)^n_{i,j+1}}{\Delta y^2}+2\xi_1\eta W^n_{i,j}. \quad (14)$$

Let us write system (14) with respect to the square of the level function, after comparison similar terms can be expressed in the form of a finite difference of a system of algebraic equations:

$$a_{i,j}h^{n+\frac{1}{2}}_{i-1,j}-b_{i,j}h^{n+\frac{1}{2}}_{i,j}+c_{i,j}h^{n+\frac{1}{2}}_{i+1,j}=-d_{i,j}, \quad (15)$$

here $a_{i,j}=\frac{2k_{i-0.5,j}\tilde{h}_{i-1,j}}{\Delta x^2}$, $b_{i,j}=\frac{2\left(k_{i-0.5,j}+k_{i+0.5,j}\right)\tilde{h}_{i,j}}{\Delta x^2}-\frac{4}{\Delta\tau}$, $c_{i,j}=\frac{2k_{i+0.5,j}\tilde{h}_{i+1,j}}{\Delta x^2}$,

$$d_{i,j}=\left(\frac{4}{\Delta\tau}-\frac{2\left(k_{i,j-0.5}+k_{i,j+0.5}\right)\tilde{h}_{i,j}}{\Delta y^2}\right)h_{i,j}^n+\frac{2k_{i,j-0.5}\tilde{h}_{i,j-1}}{\Delta y^2}h_{i,j-1}^n+$$

$$+\frac{2k_{i,j+0.5}\tilde{h}_{i,j+1}}{\Delta y^2}h_{i,j+1}^n-\frac{k_{i-0.5,j}\tilde{h}_{i-1,j}^2}{\Delta x^2}+\frac{\left(k_{i-0.5,j}+k_{i+0.5,j}\right)\tilde{h}_{i,j}^2}{\Delta x^2}-\frac{k_{i+0.5,j}\tilde{h}_{i+1,j}^2}{\Delta x^2}-$$

$$-\frac{k_{i,j-0.5}\tilde{h}_{i,j-1}^2}{\Delta y^2}+\frac{\left(k_{i,j-0.5}+k_{i,j+0.5}\right)\tilde{h}_{i,j}^2}{\Delta y^2}-\frac{k_{i,j+0.5}\tilde{h}_{i,j+1}^2}{\Delta y^2}+2\xi_1\eta W_{i,j}^{n+\frac{1}{2}}.$$

In the Oy direction, system (18) is approximated in type $\omega_{\Delta x\Delta y\Delta\tau}$ by a non-discrete scheme in a grid, using the expression for the square of the step function and expressing it as a system of three diagonal algebraic equations as follows:

$$\bar{a}_{i,j}h_{i,j-1}^{n+1}-\bar{b}_{i,j}h_{i,j}^{n+1}+\bar{c}_{i,j}h_{i,j+1}^{n+1}=-\bar{d}_{i,j}\,, \quad (16)$$

here

$$\bar{a}_{i,j}=\frac{2k_{i,j-0.5}\tilde{h}_{i,j-1}}{\Delta y^2},$$

$$\bar{b}_{i,j}=\frac{2\left(k_{i,j-0.5}+k_{i,j+0.5}\right)\tilde{h}_{i,j}}{\Delta y^2}-\frac{4}{\Delta\tau},$$

$$\bar{c}_{i,j}=\frac{2k_{i,j+0.5}\tilde{h}_{i,j+1}}{\Delta y^2},$$

Let us calculate the system of equations (16) by the sweep method:

in the direction Ox

$$h_{i,j}^{n+\frac{1}{2}}=\alpha_{i+1,j}h_{i+1,j}^{n+\frac{1}{2}}+\beta_{i+1,j}\,, \quad (17)$$

in the direction of Oy

$$h_{i,j}^{n+1}=\bar{\alpha}_{i,j+1}h_{i,j+1}^{n+1}+\bar{\beta}_{i,j+1}\,, \quad (18)$$

recurrent formulas like (17) and (18) find i replace with i -1 , also find *j* and replace j -1 :

$$h_{i-1,j}^{n+\frac{1}{2}}=\alpha_{i,j}h_{i,j}^{n+\frac{1}{2}}+\beta_{i,j}\,,$$

$$h_{i,j-1}^{n+1}=\bar{\alpha}_{i,j}h_{i,j}^{n+1}+\bar{\beta}_{i,j}$$

here sweep coefficients, after calculations *Ox, Oy* . To find the steering coefficients in the following directions, we use the following recursive expressions:

$$\alpha_i=\frac{c_{i-1,j}}{b_{i-1,j}-a_{i-1,j}\alpha_{i-1,j}},$$

$$\beta_i=\frac{d_{i-1,j}+a_{i-1,j}\beta_{i-1,j}}{b_{i-1,j}-a_{i-1,j}\alpha_{i-1,j}}, \quad (19)$$

$$\bar{\alpha}_j=\frac{\bar{c}_{i,j-1}}{\bar{b}_{i,j-1}-\bar{a}_{i,j-1}\bar{\alpha}_{i,j-1}},\ \bar{\beta}_j=\frac{\bar{d}_{i,j-1}+\bar{a}_{i,j-1}\bar{\beta}_{i,j-1}}{\bar{b}_{i,j-1}-\bar{a}_{i,j-1}\bar{\alpha}_{i,j-1}}, \quad (20)$$

$-kh\left.\frac{\partial h}{\partial n}\right|_{\Gamma}=\gamma\left(h_0-h\right)$ For the filtration zone, where the boundary condition is fictitious, we write the boundary conditions in each direction as follows and approximate by the implicit scheme:

along the route *Ox* : $$\left.\frac{\partial h}{\partial x_i}\right|_{x_i=0}=-\frac{k_0h_0}{2L}k_{1,j}\frac{2\tilde{h}_{1,j}h_{1,j}^{n+\frac{1}{2}}-\tilde{h}_{1,j}^2-2\tilde{h}_{0,j}h_{0,j}^{n+\frac{1}{2}}+\tilde{h}_{0,j}^2}{\Delta x}=\gamma(h_0h_{1,j}^{n+\frac{1}{2}}-h_0), \quad (21)$$

$$\left.\frac{\partial h}{\partial x_i}\right|_{x_i=1} = \frac{k_0 h_0}{2L} k_{I,j} \frac{2\tilde{h}_{I,j} h_{I,j}^{n+\frac{1}{2}} - \tilde{h}_{I,j}^2 - 2\tilde{h}_{I-1,j} h_{I-1,j}^{n+\frac{1}{2}} + \tilde{h}_{I-1,j}^2}{\Delta x} = \gamma(h_0 h_{I,j}^{n+\frac{1}{2}} - h_0), \quad (22)$$

on route Oy: $$\left.\frac{\partial h}{\partial y_i}\right|_{y_i=0} = -\frac{k_0 h_0}{2L} k_{i,1} \frac{2\tilde{h}_{i,1} h_{i,1}^{n+1} - \tilde{h}_{i,1}^2 - 2\tilde{h}_{i,0} h_{i,0}^{n+1} + \tilde{h}_{i,0}^2}{\Delta y} = \gamma(h_0 h_{i,1}^{n+1} - h_0), \quad (23)$$

$$\left.\frac{\partial h}{\partial y_i}\right|_{y_i=1} = \frac{k_0 h_0}{2L} k_{i,J} \frac{\tilde{h}_{i,J} h_{i,J}^{n+1} - \tilde{h}_{i,J}^2 - \tilde{h}_{i,J-1} h_{i,J-1}^{n+1} + \tilde{h}_{i,J-1}^2}{\Delta y} = \gamma(h_0 h_{i,J}^{n+1} - h_0), \quad (24)$$

As mentioned above, the problem is described using non-linear partial differential equations, which can be solved using the iterative method. Conditions for the convergence of the iterative process:

$$\left| \left(h_{i,j}\right)^{(s+1)} - \left(h_{i,j}\right)^{(s)} \right| \le \varepsilon .$$

where s is the number of iterations, ε is the iteration accuracy of the process.

Conclusions

Based on the results of the study, the following conclusions were made:

The analysis of the current state of modeling of hydrogeological processes in an arid climate, the development of modeling methods and methods for solving problems related to the interaction of ground and surface waters in multilayer porous media are analyzed. Sufficient relevance of the development of methods for the formation, forecasting and modeling of changes in river flow and its integral connection with groundwater has been revealed. As a result, the goals and objectives of the study were formed, as well as the prospects for the use of information technologies in solving the problems of rational use and management of water resources for drinking water supply.

The influence of not one, but several factors on the indicators of water infiltration in natural processes, the introduction of features and specifics of hydrogeological, hydrological, irrigation and reclamation studies into the classified hydrogeological systems, analysis of problems in non-stationary conditions, changes in the state, level, resources. Based on the actual data, the width of the river or canal is taken as the average geometric parameter along its entire length, which makes it possible to rely on the results of an experimental calculation for water consumption.

Differential equations of geofiltration processes, mathematical models, numerical solutions by the finite difference method, its algorithms and complex programming tools have been developed and implemented, taking into account the interdependence of ground and surface waters. As a result, a mathematical model of geofiltration processes was developed on the example of the Karshi groundwater reservoir, long-term plans were determined.

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